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A study has been made of the electrostatic polarization of a plasma in an applied electric field, i. e., the process of charge separation in the case when a steady current flow through the plasma is excluded.

1. Statement of the problem

Consider a plane layer of a homogeneous two-component partly ionized plasma of thickness d (Fig. 1) confined between impermeable dielectric walls $x = 0$ and $x = d$.

At $t = 0$ a constant homogeneous external electric field E_0 is applied to the plasma along the x axis.

This initiates in the plasma a movement of positive ions and electrons with respective velocities $v_p(x, t)$ and $v_e(x, t)$, which leads to deviations of the concentrations $n_p(x, t)$ and $n_e(x, t)$ from the original common level $N_p = N_e = N = \text{const}$.

Perturbations in concentration are responsible for diffusion flows due to gradients in n_p and n_e , but are also the source of an additional electric polarization field E^* , which influences the motion of the particles.

This motion of the ions and electrons is also determined by the effective frequencies of collision with neutral particles ν_p and ν_e .

If the applied field is sufficiently weak, so that all the resulting perturbations and their spatial derivatives are small, then the problem can be reduced to five linear quasihydrodynamic equations for five unknown functions: the velocities and concentrations of the ions and electrons, and electric polarization field (or potential). The temperatures of the ions T_p and electrons T_e are assumed constant.

$$\begin{aligned} \frac{\partial v_e}{\partial t} + \nu_e v_e + \frac{\kappa T_e}{mN} \frac{\partial n_e}{\partial x} &= -\frac{e}{m} (E_0 + E^*), \\ \frac{\partial v_p}{\partial t} + \nu_p v_p + \frac{\kappa T_p}{MN} \frac{\partial n_p}{\partial x} &= \frac{e}{M} (E_0 + E^*), \\ \frac{\partial v_e}{\partial x} + \frac{1}{N} \frac{\partial n_e}{\partial t} &= 0, \quad \frac{\partial v_p}{\partial x} + \frac{1}{N} \frac{\partial n_p}{\partial t} = 0, \\ \frac{\partial E^*}{\partial x} &= 4\pi e (n_p - n_e) \end{aligned} \quad (1.1)$$

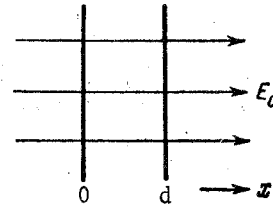


Fig. 1

Here κ is Boltzmann's constant, m and M are the mass of the electron and positive ion, respectively, and E^* is the polarization field. Neglecting the displacement current in the plasma compared with the conduction current, after a few simple transformations we get from (1.1)

$$\begin{aligned} \frac{\partial^2 v_e}{\partial t^2} + \nu_e \frac{\partial v_e}{\partial t} - \omega_e^2 \frac{\partial^2 v_e}{\partial x^2} &= -\omega_e^2 (v_e - v_p), \\ \frac{\partial^2 v_p}{\partial t^2} + \nu_p \frac{\partial v_p}{\partial t} - \omega_p^2 \frac{\partial^2 v_p}{\partial x^2} &= \omega_p^2 (v_e - v_p); \end{aligned} \quad (1.2)$$

here

$$\omega_e^2 = \frac{\kappa T_e}{m}, \quad \omega_p^2 = \frac{\kappa T_p}{M}, \quad \omega_e^2 = \frac{4\pi e^2 N}{m}, \quad \omega_p^2 = \frac{4\pi e^2 N}{M},$$

are the mean velocities of electrons and ions and their Langmuir frequencies, respectively.

System (1.2) with the corresponding boundary and initial conditions is closed and permits the unique determination of the functions $v_e(x, t)$. As boundary and initial conditions we take the relations

$$v_e(0, t) = v_p(0, t) = 0, \quad v_e(d, t) = v_p(d, t) = 0. \quad (1.3)$$

$$v_e(x, 0) = v_p(x, 0) = 0, \quad \frac{\partial v_e}{\partial t} \Big|_{t=0} = -\frac{eE_0}{m}, \quad \frac{\partial v_p}{\partial t} \Big|_{t=0} = \frac{eE_0}{M}. \quad (1.4)$$

The initial conditions imply that at $t = 0$ the velocities of the particles are still zero, but accelerations already exist, being conditioned only by the applied field, since at $t = 0$ there is no polarization field and the concentration gradients are zero.

2. Solution of system of equations (1.2)

We seek a solution in the form of series

$$v_e(x, t) = \sum_{q=1} \varphi_q(t) \sin \frac{q\pi x}{d}, \quad v_p(x, t) = \sum_{q=1} \psi_q(t) \sin \frac{q\pi x}{d} \quad (2.1)$$

satisfying boundary conditions (1.3). Using (2.1) and (1.4) we can easily find the initial conditions for the functions φ_q, ψ_q

$$\begin{aligned} \varphi_q \Big|_{t=0} = \psi_q \Big|_{t=0} &= 0, \\ \frac{d\varphi_q}{dt} \Big|_{t=0} &= \frac{2eE_0}{q\pi m} [(-1)^q - 1], \quad \frac{d\psi_q}{dt} \Big|_{t=0} = -\frac{2eE_0}{q\pi M} [(-1)^q - 1]. \end{aligned} \quad (2.2)$$

Since we are interested in a nontrivial solution, in (2.2) we must set $q = 2k + 1$. We now introduce the notations:

$$\begin{aligned} \delta_e^2 &= \omega_e^2 \left[(2k+1)^2 \pi^2 \left(\frac{h_e}{d} \right)^2 + 1 \right] - \frac{v_e^2}{4}, \quad h_e = \left(\frac{kT_e}{4\pi e^2 N} \right)^{1/2}, \\ \delta_p^2 &= \omega_p^2 \left[(2k+1)^2 \pi^2 \left(\frac{h_p}{d} \right)^2 + 1 \right] - \frac{v_p^2}{4}, \quad h_p = \left(\frac{kT_p}{4\pi e^2 N} \right)^{1/2}. \end{aligned} \quad (2.3)$$

Here h_e and h_p are the Debye shielding radii for electrons and ions. From (1.2) we obtain the equations

$$\begin{aligned} \varphi_{2k+1}'' + v_e \varphi_{2k+1}' + (\delta_e^2 + 1/4 v_e^2) \varphi_{2k+1} - \omega_e^2 \psi_{2k+1} &= 0, \\ \psi_{2k+1}'' + v_p \psi_{2k+1}' + (\delta_p^2 + 1/4 v_p^2) \psi_{2k+1} - \omega_p^2 \varphi_{2k+1} &= 0, \end{aligned} \quad (2.4)$$

where the primes denote differentiation with respect to t . The system (2.4) can be reduced by the Fubini method [1] to two integral equations

$$\begin{aligned} \varphi_{2k+1} &= -\frac{4eE_0 \exp(-1/2 v_e t)}{(2k+1)\pi m \delta_e} \sin \delta_e t + \frac{\omega_e^2}{\delta_e^2} \int_0^t \exp \frac{-v_e(t-\tau)}{2} \psi(\tau) \sin \delta_e(t-\tau) d\tau, \\ \psi_{2k+1} &= \frac{4eE_0 \exp(-1/2 v_p t)}{(2k+1)\pi M \delta_p} \sin \delta_p t + \frac{\omega_p^2}{\delta_p^2} \int_0^t \exp \frac{-v_p(t-\tau)}{2} \varphi(\tau) \sin \delta_p(t-\tau) d\tau, \end{aligned} \quad (2.5)$$

whose solution we shall obtain by the method of successive approximations. As the zero approximation it is advisable to select the first terms on the right sides of the equations

$$\varphi_{2k+1}^{(0)} = -\frac{4eE_0 \sin \delta_e t}{(2k+1)\pi m \delta_e} \exp \frac{-v_e t}{2}, \quad \psi_{2k+1}^{(0)} = \frac{4eE_0 \sin \delta_p t}{(2k+1)\pi M \delta_p} \exp \frac{-v_p t}{2},$$

which characterize the independent motion of the electrons and ions with their own characteristic times. Then

$$\begin{aligned} \varphi_{2k+1}^{(1)} &= \frac{\omega_e^2}{\delta_e^2} \int_0^t \exp \left[\frac{-v_e(t-\tau)}{2} \right] \psi_{2k+1}^{(0)}(\tau) \sin \delta_e(t-\tau) d\tau, \\ \psi_{2k+1}^{(1)} &= \frac{\omega_p^2}{\delta_p^2} \int_0^t \exp \left[\frac{-v_p(t-\tau)}{2} \right] \varphi_{2k+1}^{(0)}(\tau) \sin \delta_p(t-\tau) d\tau. \end{aligned} \quad (2.6)$$

We shall confine ourselves to the first approximation, which will enable us to take into account the interaction of the motions of electrons and ions. This effect is expressed in the fact that the motion of the electrons is characterized by the ionic, as well as electronic, characteristic times, and vice-versa.

Evaluation of the integrals (2. 6) leads to the following expressions for the first approximation:

$$\begin{aligned}\Phi_{2k+1} &= A [\exp(-\frac{1}{2}v_p t) (a_e \cos \delta_p t + b_e \sin \delta_p t) - \\ &\quad - \exp(-\frac{1}{2}v_e t) (a_e \cos \delta_e t + c_e \sin \delta_e t)], \\ \Psi_{2k+1} &= -A [\exp(-\frac{1}{2}v_e t) (a_p \cos \delta_e t + b_p \sin \delta_e t) - \\ &\quad - \exp(-\frac{1}{2}v_p t) (a_p \cos \delta_p t + c_p \sin \delta_p t)].\end{aligned}\tag{2. 7}$$

Here

$$\begin{aligned}A &= \frac{2eE_0\omega_e^2}{\delta_e\delta_p(2k+1)\pi M} = \frac{2eE_0\omega_p^2}{\delta_e\delta_p(2k+1)\pi m}, \\ a_e &= \frac{v_{ep}^-}{2} \left[\frac{1}{\frac{1}{4}(v_{ep}^-)^2 + (\delta_{pe}^+)^2} - \frac{1}{\frac{1}{4}(v_{ep}^-)^2 + (\delta_{pe}^-)^2} \right], \\ b_e &= \left[\frac{\delta_{pe}^+}{\frac{1}{4}(v_{ep}^-)^2 + (\delta_{pe}^+)^2} - \frac{\delta_{pe}^-}{\frac{1}{4}(v_{ep}^-)^2 + (\delta_{pe}^-)^2} \right], \\ c_e &= \left[\frac{\delta_{ep}^-}{\frac{1}{4}(v_{ep}^-)^2 + (\delta_{pe}^-)^2} - \frac{\delta_{pe}^+}{\frac{1}{4}(v_{ep}^-)^2 + (\delta_{pe}^+)^2} \right], \\ a_p &= \frac{v_{pe}^-}{2} \left[\frac{1}{\frac{1}{4}(v_{pe}^-)^2 + (\delta_{ep}^+)^2} - \frac{1}{\frac{1}{4}(v_{pe}^-)^2 + (\delta_{ep}^-)^2} \right], \\ b_p &= \left[\frac{\delta_{ep}^+}{\frac{1}{4}(v_{pe}^-)^2 + (\delta_{ep}^+)^2} - \frac{\delta_{ep}^-}{\frac{1}{4}(v_{pe}^-)^2 + (\delta_{ep}^-)^2} \right], \\ c_p &= \left[\frac{\delta_{pe}^-}{\frac{1}{4}(v_{pe}^-)^2 + (\delta_{ep}^-)^2} - \frac{\delta_{ep}^+}{\frac{1}{4}(v_{pe}^-)^2 + (\delta_{ep}^+)^2} \right], \\ (v_{ep}^\pm &= v_e \pm v_p, \delta_{ep}^\pm = \delta_e \pm \delta_p, v_{pe}^\pm = v_p \pm v_e, \delta_{pe}^\pm = \delta_p \pm \delta_e).\end{aligned}\tag{2. 8}$$

Thus, for $v_e(x, t)$ and $v_p(x, t)$ we have

$$\begin{aligned}v_e(x, t) &= \sum_{k=0} \left\{ -\frac{4eE_0 \exp(-\frac{1}{2}v_e t)}{(2k+1)\pi m\delta_e} \sin \delta_e t + A \left[\exp\frac{-v_p t}{2} (a_e \cos \delta_p t + \right. \right. \\ &\quad \left. \left. + b_e \sin \delta_p t) - \exp\frac{-v_e t}{2} (a_e \cos \delta_e t + c_e \sin \delta_e t) \right] \right\} \sin \frac{(2k+1)\pi x}{d}, \\ v_p(x, t) &= \sum_{k=0} \left\{ \frac{4eE_0 \exp(-\frac{1}{2}v_p t)}{(2k+1)\pi M\delta_p} \sin \delta_p t - A \left[\exp\frac{-v_e t}{2} (a_p \cos \delta_e t + \right. \right. \\ &\quad \left. \left. + b_p \sin \delta_e t) - \exp\frac{-v_p t}{2} (a_p \cos \delta_p t + c_p \sin \delta_p t) \right] \right\} \sin \frac{(2k+1)\pi x}{d}.\end{aligned}\tag{2. 9}$$

It is easy to verify that solutions (2. 9) satisfy the initial and boundary conditions (1. 3) and (1. 4).

3. Solutions for perturbations of the ion and electron concentrations

The expressions for $n_e(x, t)$ and $n_p(x, t)$ are obtained from the third and fourth of equations (1. 1), taking into account the initial conditions

$$n_e(x, t)|_{t=0} = n_p(x, t)|_{t=0} = 0.\tag{3. 1}$$

After transformation we get

$$\begin{aligned}n_e(x, t) &= N \sum_{k=0} \frac{2eE_0\omega_p^2}{\delta_e\delta_p m d} \cos \frac{(2k+1)\pi x}{d} \{ [P_e (1 - \exp(-\frac{1}{2}v_e t) \cos \delta_e t) - \\ &\quad - Q_e \exp(-\frac{1}{2}v_e t) \sin \delta_e t] - [R_e (1 - \exp(-\frac{1}{2}v_p t) \cos \delta_p t) + \\ &\quad + S_e \exp(-\frac{1}{2}v_p t) \sin \delta_p t] \}, \\ n_p(x, t) &= -N \sum_{k=0} \frac{2eE_0\omega_p^2}{\delta_e\delta_p m d} \cos \frac{(2k+1)\pi x}{d} \{ [P_p (1 - \exp(-\frac{1}{2}v_p t) \cos \delta_p t) -\end{aligned}\tag{3. 2}$$

$$- Q_p \exp(-1/2\nu_p t) \sin \delta_p t] - [R_p (1 - \exp(-1/2\nu_e t) \cos \delta_e t) + S_p \exp(-1/2\nu_e t) \sin \delta_e t]. \quad (3.3)$$

$$\begin{aligned} P_e &= \frac{2\delta_e \delta_p / \omega_p^2 + 1/2 a_e \nu_e + c_e \delta_e}{1/4 \nu_e^2 + \delta_e^2}, & P_p &= \frac{2\delta_e \delta_p / \omega_e^2 + 1/2 a_p \nu_p + c_p \delta_p}{1/4 \nu_p^2 + \delta_p^2}, \\ Q_e &= \frac{\delta_p \nu_e / \omega_p^2 - \delta_e a_e + 1/2 c_e \nu_e}{1/4 \nu_e^2 + \delta_e^2}, & Q_p &= \frac{\nu_p \delta_e / \omega_e^2 - a_p \delta_p + 1/2 c_p \nu_p}{1/4 \nu_p^2 + \delta_p^2}, \\ R_e &= \frac{1/2 a_e \nu_p + b_e \delta_p}{1/4 \nu_e^2 + \delta_e^2}, & R_p &= \frac{1/2 a_p \nu_p + b_p \delta_e}{1/4 \nu_e^2 + \delta_e^2}, \\ S_e &= \frac{a_e \delta_p + 1/2 b_e \nu_p}{1/4 \nu_p^2 + \delta_p^2}, & S_p &= \frac{a_p \delta_e - 1/2 b_p \nu_e}{1/4 \nu_e^2 + \delta_e^2}. \end{aligned}$$

4. Distribution of electron and ion concentrations in the polarized plasma

To obtain the spatial distribution of n_e and n_p after damping of all transient processes, we must take the asymptotic value of solutions (3.2) as $t \rightarrow \infty$. Here it is convenient to simplify expressions (3.3) by taking into account the obvious inequalities $\nu_e \gg \nu_p$, $\omega_e \gg \omega_p$, $\delta_e \gg \delta_p$. Then the steady-state distribution of electron and ion concentrations takes the form

$$\begin{aligned} n_e(x, \infty) &= \frac{E_0}{\pi d} \sum_{k=0}^{\infty} \frac{(2k+1)^2 \pi^2 (h_p/d)^2 \cos(2k+1)\pi x/d}{[(2k+1)^2 \pi^2 (h_e/d)^2 + 1][(2k+1)^2 \pi^2 (h_p/d)^2 + 1]}, \\ n_p(x, \infty) &= -\frac{E_0}{\pi d} \sum_{k=0}^{\infty} \frac{(2k+1)^2 \pi^2 (h_e/d)^2 \cos(2k+1)\pi x/d}{[(2k+1)^2 \pi^2 (h_e/d)^2 + 1][(2k+1)^2 \pi^2 (h_p/d)^2 + 1]}. \end{aligned} \quad (4.1)$$

The series can be summed and written in the form of a combination of hyperbolic functions:

$$\begin{aligned} n_e(x, \infty) &= \frac{E_0 h_p}{4\pi e (h_e^2 - h_p^2)} \left[\frac{\text{sh } dh_p^{-1}(1/2 - x/d)}{\text{ch } 1/2 dh_p^{-1}} - \left(\frac{T_p}{T_e}\right)^{1/2} \frac{\text{sh } dh_e^{-1}(1/2 - x/d)}{\text{ch } 1/2 dh_e^{-1}} \right], \\ n_p(x, \infty) &= \frac{E_0 h_e}{4\pi e (h_e^2 - h_p^2)} \left[\frac{\text{sh } dh_e^{-1}(1/2 - x/d)}{\text{ch } 1/2 dh_e^{-1}} - \left(\frac{T_e}{T_p}\right)^{1/2} \frac{\text{sh } dh_p^{-1}(1/2 - x/d)}{\text{ch } 1/2 dh_p^{-1}} \right]. \end{aligned} \quad (4.2)$$

In the case of a weakly ionized gas, when $h_e, h_p > d$, the expressions for $n_e(x, \infty)$ and $n_p(x, \infty)$ can be simplified:

$$\begin{aligned} n_e(x, \infty) &= \frac{E_0 d}{\pi^3 h_e^2} \sum_{k=0}^{\infty} \frac{1}{(2k+1)^2} \cos \frac{(2k+1)\pi x}{d}, \\ n_p(x, \infty) &= -\frac{E_0 d}{\pi^3 h_p^2} \sum_{k=0}^{\infty} \frac{1}{(2k+1)^2} \cos \frac{(2k+1)\pi x}{d} \end{aligned} \quad (4.3)$$

Considering the rapid convergence of the series (4.3), we can confine ourselves in the calculations for $n_e(x, \infty)$ and $n_p(x, \infty)$ to the first three terms of the series ($k=0, k=1, k=2$). The error introduced thereby does not exceed 3-4%.

5. Transient processes

The general relations (2.9) and (3.2) give the particle velocity and concentration at any point as a function of time. As in the last section, it is convenient to simplify these expressions by deliberately neglecting the small quantities ($\nu_p, \omega_p, \delta_p$) as compared with the large ($\nu_e, \omega_e, \delta_e$). In this approximation

$$v_e(x, t) = - \sum_{k=0}^{\infty} \frac{4eE_0}{(2k+1)\pi m} \left[\frac{\exp(-1/2\nu_e t) \sin \delta_e t}{\delta_e} - \frac{m/M \exp(-1/2\nu_p t) \sin \delta_p t}{\delta_p (2k+1)^2 \pi^2 (h_e/d)^2 + 1} \right] \sin \frac{(2k+1)\pi x}{d}, \quad (5.1)$$

$$v_p(x, t) = \sum_{k=0}^{\infty} \frac{4eE_0 \sin \delta_p t}{(2k+1)\pi m \delta_p} \left[1 - \frac{1}{(2k+1)^2 \pi^2 (h_e/d)^2 + 1} \right] \sin \frac{(2k+1)\pi x}{d}.$$

In the same way, for the concentrations we get

$$\begin{aligned}
 n_e(x, \infty) &= N \sum_{k=0} \frac{4eE_0\omega_p^2 \cos(2k+1)\pi x/d}{\delta_e \delta_p m d (1/4\nu_e^2 + \delta_e^2)} \left\{ \frac{\delta_p}{\omega_p^2} \left[\delta_e^2 - (1/4\nu_e^2 + \delta_e^2)^{1/2} \exp(-1/2\nu_e t) \times \right. \right. \\
 &\times \sin\left(\delta_e t + \arctg \frac{2\delta_e}{\nu_e}\right) - \frac{\delta_e}{1/4\nu_e^2 + \delta_p^2} \left[\delta_p^2 - (1/4\nu_p^2 + \delta_p^2)^{1/2} \exp(-1/2\nu_p t) \times \right. \\
 &\left. \left. \times \sin\left(\delta_p t + \arctg \frac{2\delta_p}{\nu_p}\right) \right] \right\}, \quad (5.2) \\
 n_p(x, t) &= -N \sum_{k=0} \frac{4eE_0\omega_p^2 \cos(2k+1)\pi x/d}{\delta_e \delta_p m d (1/4\nu_p^2 + \delta_p^2)} \left\{ \frac{\delta_e^2}{\omega_e^2} \frac{(2k+1)^2 \pi^2 (h_e/d)^2}{(2k+1)^2 \pi^2 (h_e/d)^2 + 1} \times \right. \\
 &\left. \times \left[\delta_p - (1/4\nu_p^2 + \delta_p^2)^{1/2} \exp(-1/2\nu_p t) \sin\left(\delta_p t + \arctg \frac{2\delta_p}{\nu_p}\right) \right] \right\}.
 \end{aligned}$$

6. Analysis of results

The solutions obtained show that the sudden application of a constant field to a plasma layer initiates in the plasma a complex motion that decays with time. After the transient processes have damped out, a state of polarization is established in the plasma; this state is a function of the intensity of the applied field and of certain plasma parameters.

Each spatial harmonic of the velocity and concentration perturbations is established with its own characteristic time. This process may take two typical forms: periodic and aperiodic. At sufficiently low gas pressures, when ν_e and ν_p are small enough for δ_e and δ_p in (2.3) to be real, the process is a damped oscillation with the two characteristic frequencies δ_e and δ_p and decay constants $2/\nu_e$ and $2/\nu_p$, respectively.

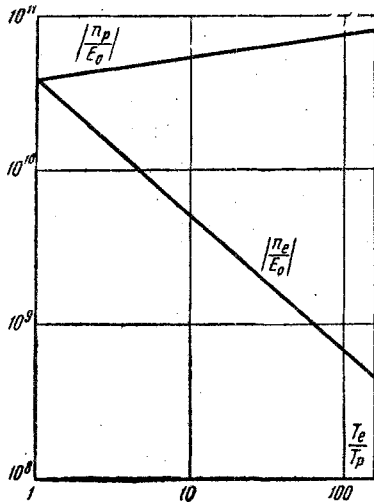


Fig. 2

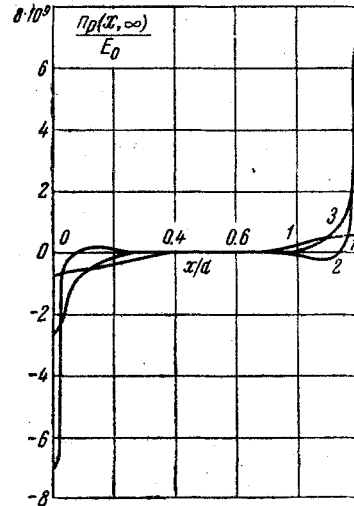


Fig. 3

In this case the spatial harmonics of the electron velocity and concentration, as is obvious from the first of equations (5.1) and (5.2), will have two characteristic frequencies and two decay constants, as distinct from the spatial harmonics of the ion velocity and concentration (second of equations (5.1) and (5.2), which have only one characteristic frequency and one decay constant. This is due to the smallness of the ratio m/M . On the other hand, at high pressures and/or low values of the concentration N , when

$$\begin{aligned}
 1/4\nu_e^2 &> \omega_e^2 [(2k+1)\pi^2 (h_e/d)^2 + 1], \\
 1/4\nu_p^2 &> \omega_p^2 [(2k+1)\pi^2 (h_p/d)^2 + 1],
 \end{aligned}$$

the settling process becomes aperiodic. In this case the phase shift can be neglected in (5.2), and in (5.1) and (5.2) we can write

$$\begin{aligned}
 F_e(t) &= \exp(-1/2\nu_e t) \sin \delta_e t = 1/2i \{ \exp[-(1/2\nu_e - \\
 &- \sqrt{1/4\nu_e^2 - \omega_e^2 [(2k+1)\pi^2 (h_e/d)^2 + 1]} t] - \\
 &- \exp[-(1/2\nu_e + \sqrt{1/4\nu_e^2 + \omega_e^2 [(2k+1)\pi^2 (h_e/d)^2 + 1]} t] \}, \quad (6.1)
 \end{aligned}$$

$$F_p(t) = \exp(-\frac{1}{2}\nu_p t) \sin \delta_p t = \frac{1}{2}i \left\{ \exp \left[-\left(\frac{1}{2}\nu_p - \sqrt{\frac{1}{4}\nu_p^2 - \omega_p^2 [(2k+1)^2 \pi^2 (h_p/d)^2 + 1]} \right) t \right] - \exp \left[-\left(\frac{1}{2}\nu_p + \sqrt{\frac{1}{4}\nu_p^2 - \omega_p^2 [(2k+1)^2 \pi^2 (h_p/d)^2 + 1]} \right) t \right] \right\} . \quad (6.1) \quad (\text{cont'd})$$

Functions F_e and F_p decay with time constants $1/\nu_e$ and $1/\nu_p$.

After an interval of several ionic mean free times, the drift velocities of the electrons and ions are practically zero, and the concentration perturbations reach their steady-state value. In this case, as can easily be seen from (4.2), the steady-state concentration distribution does not depend on the effective collision frequencies ν_e and ν_p or on the ratio m/M .

The steady-state spatial distribution of n_e and n_p is determined exclusively by the relation between the Debye radii h_e and h_p and the thickness of the plasma layer d .

It is interesting to note that for $T_e \gg T_p$ the electron concentration perturbation is negligible compared with that for the ions. This is plainly visible in Fig. 2 which shows n_p and n_e in the polarized plasma in the immediate vicinity of the wall as functions of T_e/T_p , as calculated (in CGSE units) for the case: $d = 1$ cm, $N = 10^{10}$ cm $^{-3}$, $T_p = 1000^\circ$ K. This result fully agrees with the previously published [2] qualitative argument concerning the predominant role played by the ions in the process of electrostatic plasma polarization when $T_e \gg T_p$.

As may be seen from Fig. 2, the ratio n_p/n_e in a polarized plasma is of the same order as T_e/T_p .

Figure 3 shows the distribution of the ion concentration perturbation in a polarized plasma for three cases, with the initial level of electron and ion concentration as parameter. The calculations have been made for the case: $d = 1$ cm, $T_p = 1000^\circ$ K, $T_e = 100\,000^\circ$ K. As may be seen from Fig. 3, with increase in concentration level the perturbation is concentrated in ever thinner layers near the walls.

At $N 10^6$ cm $^{-3}$ the concentration of ions is perturbed practically throughout the plasma layer, since we then have $h_e > d$.

The above-mentioned characteristic times of the transient processes for electrostatic plasma polarization can be used for plasma diagnostics. In this respect, the characteristic times of the ions are of particular interest, since measurement of the parameters of the ionic component involves known difficulties.

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